

Probability and Probability Distributions

Exercise 1:

What is the probability of getting an ace or a club on a single pick from a well-shuffled standard deck of cards? (In all remaining problems, it will be implicitly assumed that coins are balanced, die are fair, and decks of cards are standard and well-shuffled, and cards are picked at random without replacement)

Solution:

Getting an ace or a club does not constitute two mutually exclusive events because we could get the ace of clubs. Applying the rule of addition for events that are not mutually exclusive, we get

$$p(A \text{ or } C) = p(A) + p(C) - p(A \text{ and } C) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

The preceding probability statement can be rewritten in an equivalent way using set theory as follows:

$$p(A \cup C) = p(A) + p(C) - p(A \cap C)$$

Where \cap is the intersection symbol and *stands for and*.

Exercise 2:

What is the function of the negative term in the rule of addition for events that are not mutually exclusive?

Solution:

The function of the negative term in the rule of addition for events that are not mutually exclusive is to avoid double counting. For example in calculating $p(A \text{ or } C)$ in example 5a, the ace of clubs is counted twice, once as an ace and once as a club.

Therefore, we subtract the probability of getting the ace of clubs in order to avoid this double counting. If the events are mutually exclusive, the probability that both events will occur simultaneously is 0, and no double counting is involved. This is why the rule of addition for mutually exclusive events does not contain a negative term.

Exercise 3:

What is the probability of : (a) Inflation, I, or recession, R if the probability of inflation is 0.3, the probability of recession is 0.2 and the probability of inflation and recession is 0.06? (b)

Since the probability of : (a) Inflation, I or recession, R If the probability of inflation is 0.3, the probability of recession is 0.2 and the probability of inflation and recession is 0.06

Answer: since the probability of inflation and recession is not 0. Inflation and recession are not mutually exclusive events. Applying the rule of addition, we get

$$p(I \text{ or } R) = p(I) + p(R) - p(I \text{ and } R)$$

$$\text{Or } p(I \cup R) = p(I) + p(R) - p(I \cap R)$$

$$\text{and } p(I \text{ or } R) = p(I \cup R) = 0.3 + 0.2 - 0.06 = 0.44$$

Exercise 4:

Whats is probability of drawing an ace or a club, or a diamond on a single pick from a deck?

Solution:

Getting an ace, a club, or a diamond does not constitute mutually exclusive events because we could get the ace of clubs or the ace of diamonds, Applying the rule of addition for events that are not mutually exclusive, we get

$$P(A \text{ or } C \text{ or } D) = p(A) + p(C) + p(D) - p(A \text{ and } C) - p(A \text{ and } D)$$

$$P(A \text{ or } C \text{ or } D) = \frac{4}{52} + \frac{13}{52} + \frac{13}{52} - \frac{1}{52} - \frac{1}{52} = \frac{28}{52} = \frac{7}{13}$$